Factorizaciones en $\mathbb{Z}[X]$

$$
\begin{aligned}
X^2-1 &= (X-1)(X+1),\\
X^3-1 &= (X-1)(X^2+X+1),\\
X^4-1 &= (X-1)(X+1)(X^2+1),\\
X^5-1 &= (X-1)(X^4+X^3+X^3+X+1).\\
X^6-1 &= (X-1)(X+1)(X^2+X+1)(X^2-X+1),\\
X^7-1 &= (X-1)(X^6+X^5+X^4+X^3+X^2+X+1),\\
X^8-1 &= (X-1)(X+1)(X^2+1)(X^4+1),\\
X^9-1 &= (X-1)(X^2+X+1)(X^6+X^3+1),\\
X^{10}-1 &= (X-1)(X+1)(X^4+X^3+X^2+X+1)(X^4-X^3+X^2-X+1).
\end{aligned}
$$

Factorizaciones en $\mathbb{F}_p[X]$

$\underline{X^2-1}$

$p=2$:	$(X+1)^2$
$p=3$:	$(X-1)(X+1)$
$p=5$:	$(X-1)(X+1)$
$p=7$:	$(X-1)(X+1)$
$p=11$:	$(X-1)(X+1)$

$\underline{X^3-1}$

$p=2$:	$(X+1)(X^2+X+1)$
$p=3$:	$(X-1)^3$
$p=5$:	$(X-1)(X^2+X+1)$
$p=7$:	$(X-1)(X-2)(X-4)$
$p=11$:	$(X-1)(X^2+X+1)$

$\underline{X^4-1}$

$p=2$:	$(X+1)^4$
$p=3$:	$(X-1)(X+1)(X^2+1)$
$p=5$:	$(X-1)(X-2)(X-3)(X-4)$
$p=7$:	$(X-1)(X+1)(X^2+1)$
$p=11$:	$(X-1)(X+1)(X^2+1)$

$\underline{X^5-1}$

$p=2$:	$(X+1)(X^4+X^3+X^2+X+1)$
$p=3$:	$(X-1)(X^4+X^3+X^2+X+1)$
$p=5$:	$(X-1)^5$
$p=7$:	$(X-1)(X^4+X^3+X^2+X+1)$
$p=11$:	$(X-1)(X-3)(X-4)(X-5)(X-9)$

$\underline{X^6-1}$

$p=2$:	$(X+1)^2(X^2+X+1)^2$
$p=3$:	$(X+1)^3(X+2)^3$
$p=5$:	$(X-1)(X+1)(X^2+X+1)(X^2-X+1)$
$p=7$:	$(X-1)(X-2)(X-3)(X-4)(X-5)(X-6)$
$p=11$:	$(X-1)(X+1)(X^2+X+1)(X^2-X+1)$

$\underline{X^7-1}$

$p=2$:	$(X+1)(X^3+X+1)(X^3+X^2+1)$
$p=3$:	$(X-1)(X^6+X^5+X^4+X^3+X^2+X+1)$
$p=5$:	$X^6+X^5+X^4+X^3+X^2+X+1$
$p=7$:	$(X-1)^7$
$p=11$:	$(X-1)(X^3+5X^2+4X-1)(X^3+7X^2+6X-1)$

$\underline{X^8-1}$

$p=2$:	$(X+1)^8$
$p=3$:	$(X-1)(X+1)(X^2+1)(X^2+X-1)(X^2-X-1)$
$p=5$:	$(X-2)(X-3)(X-4)(X^2-2)(X^2-3)$
$p=7$:	$(X-1)(X+1)(X^2+1)(X^2+4X+1)(X^2-4X+1)$
$p=11$:	$(X-1)(X+1)(X^2+1)(X^2+3X-1)(X^2-3X-1)$

$\underline{X^9-1}$

$p=2$:	$(X+1)(X^2+X+1)(X^6+X^3+1)$
$p=3$:	$(X-1)^9$
$p=5$:	$(X-1)(X^2+X+1)(X^6+X^3+1)$
$p=7$:	$(X-1)(X-2)(X-1)(X^3-2)(X^3-4)$
$p=11$:	$(X-1)(X^2+X+1)(X^6+X^3+1)$

$\underline{X^{10}-1}$

$p=2$:	$(X+1)^2(X^4+X^3+X^2+X+1)^2$
$p=3$:	$(X-1)(X+1)(X^4+X^3+X^2+X+1)(X^4-X^3+X^2-X+1)$
$p=5$:	$(X-1)^5(X+1)^5$
$p=7$:	$(X-1)(X+1)(X^4+X^3+X^2+X+1)(X^4-X^3+X^2-X+1)$
$p=11$:	$(X-1)(X-2)(X-3)(X-4)(X-5)(X-6)(X-7)(X-8)(X-9)(X-10)$